

Takeover Bids below the Expected Value of Minority Shares

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Abstract

Focusing on takeover bids for which the outcome can be predicted in advance with certainty, Grossman and Hart established the proposition, which subsequent work accepted, that successful bids must be made at or above the expected value of minority shares. This proposition provided the basis for Grossman and Hart's identification of a free-rider problem and became a major premise for the analysis of takeovers. This paper shows that this important proposition does not always hold once we drop the assumption that the only successful bids are those whose success could have been predicted with certainty. In particular, it is shown that any unconditional bid that is below the expected value of minority shares but above the independent target's per share value will succeed with a positive probability, that the bidder's expected payoff from such a bid (not counting the transaction costs of making the bid) is always positive, and that bidders might elect to make such bids. These results have implications for the nature of the free-rider problem and for the operation of takeovers; in particular, it is shown that, when a raider can increase the value of a target's assets, the raider might elect to bid even if no dilution of minority shares is possible and it holds no initial stake in the target.

I. Introduction

In an already classic paper, Grossman and Hart (1980) advanced the proposition, which subsequent work accepted, that successful takeover bids must be made at or above the expected value of minority shares. This proposition led to Grossman and Hart's insightful observation that a free-rider problem exists, and it was used by them and subsequently by others to analyze the operation of takeovers. This paper shows that this important proposition does not always hold once we drop Grossman and Hart's assumption that the only successful bids are those for which success could have been predicted with certainty. It is shown that bids below the expected value of minority shares may succeed with a positive probability, that such bids may be profitable and may be used by bidders, and that these possibilities have implications for the nature of the free-rider problem and for the operation of takeovers.

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Suppose that the value of a target's assets is V_o per share under the present management, and that, under the control of a given raider, the value of these assets would increase to V_i per share. Suppose also that the expected value of minority shares in the event of a takeover, V_m , exceeds V_o , because expropriation of the value of minority shares would either be impossible or at least possible only to a limited extent. Grossman and Hart, assuming that the only successful bids are those whose success can be predicted with certainty, showed that a takeover bid by the raider at any price below V_m cannot succeed. A success with certainty, they pointed out, cannot be a rational expectations outcome of a bid below V_m : supposing that a takeover is going to take place, each atomistic shareholder will prefer to hold out and end up with a minority share worth V_m rather than have his share acquired for the lower bid price. Thus, minority shareholders' ability to free ride on the raider's improvement (at least in part) would rule out a takeover at any price below V_m , even if this offered price exceeds the target's independent value V_o .

From their conclusion that only bids at or above V_m can succeed, Grossman and Hart drew an important implication concerning the special case in which no expropriation is possible (i.e., $V_m = V_i$). In this case, Grossman and Hart suggested, the raider would not be able to make any gain on shares acquired through a successful takeover bid. Therefore, since there are costs to making a bid, the raider would not bid, and the absence of expropriation would thus prevent a value-increasing takeover from occurring.

The subsequent work on takeovers has accepted—and recognized the significance of—the Grossman and Hart conclusion that bids below the expected value of minority shares cannot succeed (see, e.g., Fishman (1988), Harris and Raviv (1988), Hirshleifer and Titman (1988), Jegadeesh and Chowdhry (1988), Khanna (1988), Scharfstein (1988), and Shleifer and Vishny (1986)). The free-rider corollary—that, without expropriation, a raider would be unable to profit on shares acquired through a bid—motivated, and served as a useful starting point for, an examination of what explains the making of takeover bids. Grossman and Hart suggested that an incentive to bidding is created because legal rules and charter provisions enable acquirers to expropriate some of the value of minority shares, and analysts of takeover bids have generally followed this approach and assumed the presence of a sufficient level of expropriation (see, e.g., Fishman (1988), Khanna (1988), and Scharfstein (1988)). Shleifer and Vishny (1986)—and, subsequently, Jegadeesh and Chowdhry (1988) and Hirshleifer and Titman (1988)—analyzed the incentive to bidding created by an initial stake held by the raider; an acquirer that held an initial stake in the target prior to bidding will gain from the appreciation of its initial stake even if it makes no gain on shares acquired through its bid.

Furthermore, the Grossman and Hart proposition has been an important premise of subsequent models of the case in which some expropriation is possible (which, as just noted, has been generally regarded to be the common case). Analysts modelling the operation and outcome of bids have generally assumed that bids below the expected value of minority shares cannot succeed (see, e.g., Scharfstein (1988) and Harris and Raviv (1988)).

This paper drops the Grossman and Hart assumption that the only successful

bids are those for which success could have been predicted with certainty, and it then reconsiders the prospects, profitability, and use of bids below V_m . It is shown that any bid below V_m but above V_o may succeed if the bid is unconditional—that is, a bid committing the bidder to purchase tendered shares even if the bid fails. The intuition behind this result can be stated briefly as follows. In the case of an unconditional bid below V_m but above V_o , although certain success of the bid is not a rational equilibrium outcome, neither is a certain failure. Nontendering is not an equilibrium strategy for the target's shareholders because, if other shareholders are going to hold out and the bid is going to fail, each atomistic shareholder will prefer to tender and have his share acquired for a price exceeding the target's independent value V_o . Such an unconditional bid has a unique symmetric equilibrium, which we identify and characterize, in which shareholders use mixed strategies and the bid may consequently either succeed or fail.

Moreover, we show that unconditional bids below V_m but above V_o may be profitable and, consequently, may be used by bidders. Although such a bid would produce a loss if the bid fails and the bidder must purchase shares at a price exceeding V_o , the expected payoff from the bid, exclusive of the transaction costs of making the bid, is shown to be always positive. To examine when such bids will be used, we seek to identify which bid below V_m will produce the highest expected payoff, and to identify the conditions under which this expected payoff will exceed both the costs of making the bid and the expected payoff from bidding at or above V_m . In particular, we show that, in the absence of expropriation, a raider that can increase the value of the target's assets may bid even if it holds no initial stake in the target, and that, in the presence of some expropriation, a bidder may elect to bid below V_m , even though such a bid would not succeed with certainty as would a bid at V_m .¹

Section II analyzes the case in which no expropriation is possible and Section III analyzes the case in which some expropriation is possible. Section IV then makes concluding remarks concerning the implications of the paper's analysis.

II. Bidding when No Expropriation Is Possible

A. Assumptions

The value of the target's assets under existing management is V_o per share. If another agent—which shall be called “the raider”—gains control over the target, the per share value of its assets will be $V_t > V_o$. $I = V_t - V_o$ is the per share improvement that a takeover by the raider can produce. It will be initially

¹ With respect to the no-expropriation case, some of this paper's results overlap with those reached independently by Bagnoli and Lipman (1988). Bagnoli and Lipman show that, in the absence of expropriation, an unconditional bid below V_m may succeed. But they do not identify, as does this paper, which such bid will produce the highest payoff to the bidder. Furthermore, Bagnoli and Lipman do not examine the outcome of bids below V_m in the case in which some expropriation is possible, and they thus do not show that unconditional bids below V_m may succeed in this case and, moreover, may be optimal for bidders to use. (In examining the case in which some expropriation is possible, Bagnoli and Lipman focus on a quite different issue, viz. the possibility that a bidder who is less efficient than current management can take the company over profitably (with a bid below V_o and V_t but above V_m .)

assumed that, in the event of a takeover, the bidder will be unable to dilute the value of minority shares. Thus, minority shares will have a per share value of $V_m = V_r$.

The target has N shares, where N is large. The raider will have control if and only if it owns at least kN shares, where $0 < k < 1$ indicates the fraction of shares necessary for control. The raider is assumed, for simplicity of exposition, to hold no initial stake in the target. (The analysis can be adjusted to apply to the case in which the raider owns initially some noncontrolling fraction s ($s < k$) of the target's shares.)

Each share of the target is held by one shareholder. Because N is large, it is assumed, like Grossman and Hart, that the atomistic shareholders ignore the possibility that their decision will determine the outcome of a bid. Dropping this assumption would make the analysis more complicated, but would not change its conclusions.

It is assumed, as in Grossman and Hart, that, if the raider does bid and its bid fails, the per share value of the target's shares will be the status quo value of V_o . That is, should the bid fail, investors will not expect the failing bidder (or another bidder) to make a subsequent bid; if the raider's failing bid requires it to purchase some shares, the raider is expected to (or even has committed itself to) sell those shares on the market or hold them as a noncontrolling block.

In the event that the raider elects to bid, B will denote the bid price. For any bid B , b will denote the value of $(B - V_o)/I$; that is, b is the fraction of the takeover improvement that is offered by the raider as a premium. T will denote the number of shares attracted by a bid. Two kinds of bids shall be considered: conditional, where the raider is willing to buy shares only if $T \geq kN$, and unconditional, where no such condition is attached. The total cost to the raider of making a bid is C . The raider, of course, will elect to bid only if the expected profit on shares purchased through the bid exceeds C .

It is easy to see that, although a bid with $B \geq V_r$ would succeed, it would not provide the raider with any gain on shares purchased through the bid. Thus, we should consider whether the bidder can make a gain by bidding below V_r . In particular, since it is clear that any bid below V_o would attract no shares, the focus should be on bids that are below V_r but not below V_o .

B. Conditional Bid below V_r Cannot Succeed

It is worth starting the analysis with conditional bids, the case in which the proposition that bids below V_r cannot succeed does always hold. Consider a bid with $V_o < B < V_r$ that is conditional on $T \geq kN$. In this situation, the dominant strategy of each atomistic shareholder will be not to tender. In the event that the bid fails, the shareholder's tender decision will not matter; whether he tenders or holds out, his share will not be acquired. In the event that the bid succeeds, nontendering (and thus ending up with V_r) is clearly preferable to tendering (and having one's share acquired for $B < V_r$). Thus, we have the following proposition.

Proposition 1. Any bid $V_o < B < V_r$ ($0 < b < 1$) that is conditional on the bidder's gaining control ($T \geq kN$) will fail.

C. Unconditional Bids below V_t May Succeed

Let us now consider the outcome of a bid $V_o < B < V_t$ that is unconditional. In this situation, there is no equilibrium in pure strategies. Nontendering is no longer an equilibrium strategy (as it was in the case of a conditional bid). For, assuming that other shareholders are going to hold out, a shareholder will not be indifferent between tendering and holding out (as he was in the case of a conditional bid): he will prefer to tender and have his share acquired for B rather than remain with a share in the independent target worth V_o .

As to tendering, it is not an equilibrium strategy for the same reasons as in the case of a conditional bid. Assuming that others are going to tender and a takeover is going to take place, a shareholder will prefer to hold out and end up with a minority share worth V_t rather than tender and have his share acquired for the lower bid price of B .

It remains then to consider the possibility of an equilibrium in mixed strategies. Focusing on the possibility of a symmetric equilibrium, let us consider the shareholder strategy of tendering with probability t and holding out with probability $1-t$, where $0 < t < 1$. If all the shareholders follow this strategy, the likelihood of a takeover will be

$$(1) \quad Pr(t) = 1 - F(N, t, kN - 1) = \sum_{j=kN}^N \frac{N!}{j!(N-j)!} t^j (1-t)^{N-j},$$

where $F(x, y, z)$ is the binomial distribution function, indicating the probability that out of x independent trials, each with probability y of success, no more than z will be successful.

Clearly, for t to define an equilibrium strategy, a given shareholder must be indifferent between tendering and holding out, given the probability $Pr(t)$ that the bid will succeed. In comparison to nontendering, tendering produces a gain of $(B - V_o)$ in the event that the bid fails, and a loss of $(V_t - B)$ in the event that the bid succeeds. That is, t must satisfy

$$(2) \quad (B - V_o) [1 - Pr(t)] - (V_t - B) [Pr(t)] = 0,$$

or, rearranging terms,

$$(3) \quad Pr(t) = \frac{B - V_o}{V_t - V_o} = b.$$

It can be easily seen that $Pr(t)$ goes to zero as t goes to zero, that $Pr(t)$ goes to 1 as t goes to 1, and that $Pr(t)$ increases monotonically in the interval $(0, 1)$. Thus, for any $V_o < B < V_t$, there is exactly one value of t , let us denote it by τ , which satisfies (3). We can thus state the following proposition.

Proposition 2. For any unconditional bid $V_o < B < V_t$, there is a unique symmetric equilibrium, in which shareholders tender with probability $\tau(B)$, satisfying

$$(4) \quad Pr(\tau) = \frac{B - V_o}{V_t - V_o},$$

and in which the bid consequently succeeds with probability

$$(5) \quad Pr(\tau) = b = (B - V_o) / (V_t - V_o) .$$

Remarks. (i) An increase in the Bid Price B will raise both the likelihood of a takeover $Pr(\tau)$ and the equilibrium probability of tendering τ . $Pr(\tau)$ will increase because it will always equal b , the fraction of the improvement that the raider's premium is offering. τ will increase to produce the necessary increase in $Pr(\tau)$. Both $Pr(\tau)$ and τ will go to 1 as B goes to V_t , and both will go to 0 when B goes to V_o .

(ii) An increase in the control fraction k will not affect the value of $Pr(\tau)$, but it will raise τ . Since $Pr(\tau)$ is always equal to b , it is independent of the value of other parameters. The value of τ will increase, however, to maintain this necessary equality between $Pr(\tau)$ and b ; as k increases, the τ that produces a given value of $Pr(\tau)$ increases as well.

D. The Expected Payoff from Bidding below V_t

When a bidder makes an unconditional bid $V_o < B < V_t$, there are two possible outcomes. The bid may succeed in attracting kN shares or more, in which case the bidder will gain $(V_t - B)$ for each tendered share. Or the bid may fail, in which case the bidder will lose $(B - V_o)$ for each tendered share. Thus, the bidder's expected payoff (not counting the costs of making the bid) will be

$$(6) \quad W(B) = -(B - V_o) \sum_{j=1}^{kN-1} f(N, \tau, j)j + (V_t - B) \sum_{j=kN}^N f(N, \tau, j)j ,$$

where $f(x, y, z)$ is the binomial density function, indicating the likelihood that out of x independent experiments, each with probability of success y , there will be exactly z successes.

Since N is large, the binomial distribution can be well approximated using the normal distribution (see Feller, Ch. 7). In particular, $W(B)$ can be well approximated by

$$(7) \quad W(B) = -(B - V_o) \int_{-\infty}^{q(\tau)} \psi(x) [x\sqrt{N\tau(1-\tau)} + N\tau] dx + (V_t - B) \int_{q(\tau)}^{\infty} \psi(x) [x\sqrt{N\tau(1-\tau)} + N\tau] dx ,$$

where $\psi(\cdot)$ is the standard normal density function and

$$(8) \quad q(\tau) = \frac{kN - \tau N}{\sqrt{N\tau(1-\tau)}} .$$

Proposition 3. The expected payoff to the bidder (not counting the costs of making the bid) from making any unconditional bid $V_o < B < V_t$ is positive.

Remark. The result of Proposition 3 can be intuitively explained as follows. If

the bidder expected to purchase the same number of shares whether the bid succeeds or fails, then his expected payoff, per share to be purchased, would be $Pr(\tau)(V_t - B) - [1 - Pr(\tau)](B - V_o)$. But that τ is an equilibrium strategy (i.e., that shareholders are indifferent between tendering and holding out) implies that this payoff would be zero. Thus, because the bidder can expect to purchase more shares in the event the bid succeeds ($T \geq K$), than in the event the bid fails ($T < K$), it follows that the bidder faces a positive expected payoff.

Proof. Rewriting (7), we get

$$(9) \quad W(B) = -(B - V_o) \int_{-\infty}^{\infty} \psi(x) [x\sqrt{N\tau(1-\tau)} + N\tau] dx \\ + (V_t - V_o) \int_{q(\tau)}^{\infty} \psi(x)x\sqrt{N\tau(1-\tau)} dx \\ + (V_t - V_o) \int_{q(\tau)}^{\infty} \psi(x)N\tau dx .$$

The first expression on the right-hand side of (9) can be shown to equal $-(B - V_o)N\tau$. The third expression on the right-hand side can be shown to equal $(V_t - V_o)Pr(\tau)N\tau$. Thus, by the condition that τ is the equilibrium probability of tendering and thus satisfies $Pr(\tau) = (B - V_o)/(V_t - V_o)$ (see (5)), the first and third expressions add up to zero. Thus, the value of $W(B)$ is equal to the value of the second expression. Doing the integration in that expression, we get

$$(10) \quad W(B) = \sqrt{N\tau(1-\tau)/2\pi} e^{-1/2|q(\tau)|^2} I ,$$

and the right-hand side of (10) is clearly positive.

E. The Optimal Bid

Supposing that the raider is going to make an unconditional bid $V_o < B < V_t$, its optimal bid is given by the solution to the problem

$$(11) \quad \max_B W(B) .$$

Solving this maximization problem provides the following proposition, which is proven in the Appendix.

Proposition 4. The optimal bid is one offering a premium equal to half of the potential improvement ($B = (V_t + V_o)/2$). In the face of this bid, each shareholder will tender with probability equal to the control fraction ($\tau = k$), and the bid will succeed with likelihood $1/2$.

F. The Incentive to Bid

We can now determine when the raider will elect to bid (even though there is no expropriation). If the raider does bid, it will offer, by Proposition 4, a price

of $(V_t + V_o)/2$. Substituting this value for B in (10) and making the calculation shows that the raider's expected payoff in such a case will be

$$(12) \quad W \left[\frac{V_t + V_o}{2} \right] = \sqrt{Nk(1-k)/2\pi} I.$$

Since the raider will bid if and only if the expected payoff given by (12) exceeds the cost C of making the bid, we have the following proposition.

Proposition 5. In the absence of expropriation, the raider will bid for the target if and only if the ratio of the costs of making the bid to the total improvement produced by a takeover satisfies

$$(13) \quad \frac{C}{NI} < \sqrt{k(1-k)/2\pi N}.$$

For example, supposing that $k = 1/2$. Then, for $N = 500$, a bid will take place if and only if C does not exceed 1.8 percent of the total improvement to be produced; for $N = 1,000$, a bid will take place if and only if C does not exceed 1.25 percent of this total improvement; and, for $N = 5,000$, a bid will take place if and only if C does not exceed 0.55 percent of the total improvement.²

III. Bidding when Some Expropriation Is Possible

Let us now consider the case in which expropriation of the value of minority shares would be possible to some extent. Specifically, suppose that in the event of a takeover the raider would be able to dilute the value of each minority share by an amount D , where $I > D > 0$. Thus, $(I - D)$ is that part of the improvement produced by a takeover that the raider would be unable to deny minority shareholders, and on which these shareholders could thus free ride. Thus, the value of minority shares in the event of a takeover will be $V_m = V_t - D$, where $V_o < V_m < V_t$.³

As before, the interest is in examining bids with $V_o < B < V_m$. In particular, we shall examine whether such bids may succeed and whether raiders will choose to make such bids.

A. The Outcome of Bids below V_m

The analysis of the outcome of a bid below V_m can proceed in the same way as in Section II. The analysis of bid outcomes in Section II depended only on the relationship between B , V_o , and the expected value of minority shares V_m ; V_t appeared in that analysis only because, given the assumption of no expropriation, V_m was equal to V_t .

² That such numbers might be interesting to look at is suggested by the information contained in the CDE Stock Ownership Directory. Looking at a random sample of 30 Fortune-500 companies, we found that each of these companies had, on average, 41.6 shareholders with a block exceeding 0.2 percent of the total stock—and that the cumulative holdings of these shareholders averaged 55 percent of the company's total stock.

³ Because the concern of this paper is with bids below the expected value of minority shares, there is no interest in analyzing the obvious case in which $D > I$ and hence $V_m < V_o$. In this case, a bid below V_m cannot succeed. Note, however, that in such a case, a bid below V_o may succeed.

A bid below V_m that is conditional on the bidder's gaining control is bound to fail, for the same reasons as those identified in Section II. Nontendering is a dominant strategy, because (i) assuming that the bid is going to fail (and tendered shares are consequently going to be returned), a shareholder will be indifferent between tendering and nontendering, and (ii) assuming that the bid is going to succeed, a shareholder will prefer to remain with minority shares worth V_m rather than to have his share purchased for a price below that value. Thus, we have Proposition 6.

Proposition 6. Any bid with $B < V_m$ that is conditional on the bidder's gaining control ($T \geq kN$) will fail.

In the case of an unconditional bid below V_m , there is no equilibrium in pure strategies, for the same reasons as those discussed in Section II. Supposing that other shareholders are going to tender, a shareholder will prefer to hold out; and supposing that other shareholders are going to hold out, a shareholder will prefer to tender. Proceeding in the same way as in Section II, we find that there is a unique symmetric equilibrium in mixed strategies that is characterized by the following proposition.

Proposition 7. For any unconditional bid $V_o < B < V_m$, there is a unique symmetric equilibrium, in which shareholders tender with probability $\tau(B)$ satisfying

$$(14) \quad 1 - F(N, \tau, kN - 1) = \frac{B - V_o}{V_m - V_o},$$

with $F(\cdot, \cdot, \cdot)$ indicating the binomial distribution function, and in which the bid succeeds with probability

$$(15) \quad Pr(\tau) = \frac{B - V_o}{V_m - V_o}.$$

Thus, the likelihood that an unconditional bid is going to succeed is equal to the ratio of the premium offered to that part of the improvement that minority shareholders can capture.

B. The Expected Payoff from Bidding below V_m

When a bidder makes an unconditional bid $V_o < B < V_m$, there are two possible outcomes. If the bid fails, which has a likelihood of $(V_m - B)/(V_m - V_o)$, the bidder will lose $(B - V_o)$ on each tendered share. If the bid succeeds, which has a likelihood of $(B - V_o)/(V_m - V_o)$, the bidder will make a gain of $(V_t - B)$ on each tendered share and, in addition, a gain of D on each nontendered share. Since $V_t - B = D + (V_m - B)$, the bidder's gain, in the event of a takeover, can be alternatively described as a gain of D on all of the target's shares and, in addition, a gain of $(V_m - B)$ on each tendered share. Thus, the bidder's expected payoff from the bid (not counting the costs of making the bid) will be

$$(16) \quad W(B) = -(B - V_o) \sum_{j=1}^{kN-1} f(N, \tau, j)j + (V_t - B) \sum_{j=kN}^N f(N, \tau, j)j + DN \frac{B - V_o}{V_m - V_o},$$

where, as before, $f(\cdot, \cdot, \cdot)$ is the binomial density function. Again, using the normal approximation, we can get

$$(17) \quad W(B) = -\left(B - V_0\right) \int_{-\infty}^{q(\tau)} \psi(x) \left[x \sqrt{N\tau(1-\tau)} + N\tau \right] dx \\ + \left(V_m - B\right) \int_{q(\tau)}^{\infty} \psi(x) \left[x \sqrt{N\tau(1-\tau)} + N\tau \right] dx + DN \frac{B - V_o}{V_m - V_o},$$

where, as before, $\psi(\cdot)$ is the standard normal density function.

Noting the similarity between the first two expressions on the right-hand side of (17) and the two expressions on the right-hand side of (7), we can follow the steps taken in proving Proposition 3 to get

$$(18) \quad W(B) = \sqrt{N\tau(1-\tau)/2\pi} e^{-1/2[q(\tau)]^2} (V_m - V_o) + D \frac{B - V_o}{V_m - V_o},$$

and since the first expression on the right-hand side of (18) is clearly positive, we have the following proposition.

Proposition 8. The expected payoff to the bidder (not counting the costs of making the bid) from making any unconditional bid $V_0 < B < V_m$ is greater than $DN(B - V_0)/(V_m - V_0)$.

C. The Possible Use of Bids below V_m

In the case in which no expropriation is possible, the only two options of the bidder that required our consideration were to bid below V_m and to not bid; for, in the absence of expropriation, the bidder could not make any gain on shares acquired through a bid at or above $V_m = V_t$. Thus, the bidder's choice was determined by comparing the maximum expected payoff from bidding below V_m with the cost of making a bid. In the case in which expropriation is possible, however, a bid at $V_m = V_t - D$ will provide the bidder with a gain of DN (not counting the costs of making the bid). Thus, it is necessary to examine whether a bid below V_m will ever be more profitable than a bid at V_m . As will be seen below, the answer to this question is affirmative.

Assessing the consequences of lowering its bid from V_m , a bidder will have to consider two effects. On the one hand, while a bid at V_m is bound to succeed, the lower bid may fail; in this case, lowering the bid would produce a loss, both because the bidder would lose on shares purchased through the failing bid and because the bidder would lose the potential gain of DN that bidding at V_m could produce. On the other hand, the lower bid may succeed; in this case, lowering the bid by x would save x on each share purchased.

We have seen that, in the no-expropriation case of $D = 0$, $V_m = V_t$, it would be indeed optimal for the bidder to lower the bid from V_m to $(V_m + V_o)/2$. In comparison to this no-expropriation case, the presence of expropriation increases the cost of lowering the bid from V_m : in the presence of expropriation, a bid at V_m would produce a gain of DN , and any lowering of the bid from this level would increase the likelihood of losing this potential gain. Thus, it seems

reasonable to suspect that whether the optimal bid would be lower than V_m —and, if so, by how much—would depend on the size of D relative to I . In particular, it seems reasonable to suspect that increasing D makes it less likely that the optimal bid will be below V_m or, if the optimal bid is below V_m , less likely that the optimal bid fall below V_m by a substantial margin. And indeed, such a relationship is suggested by the following two propositions, which are proven in the Appendix.

Proposition 9. Supposing that the raider is going to bid, a sufficient condition for the optimal bid to be below V_m is that D satisfy

$$(19) \quad D < \frac{1-k}{2-k} I.$$

Proposition 10. Supposing that the raider is going to bid, a sufficient condition for the optimal bid to be above $V_m - \alpha D$ is that D satisfy

$$(20) \quad D \geq \frac{[k(1-k)/2\pi\alpha^2N]^{1/4}}{1 + [k(1-k)/2\pi\alpha^2N]^{1/4}} I.$$

To illustrate, suppose for example that $k = 0.50$ and that $N = 1,000$. Then, by Proposition 9, a sufficient condition for the optimal bid to be below V_m is that D is no greater than $0.33I$. And, by Proposition 10, a sufficient condition for the optimal bid to exceed $V_m - D/4$ is that D is not lower than $0.15I$.

IV. Concluding Remarks

This paper has extended the Grossman and Hart analysis to allow for bids for which the outcome cannot be predicted in advance with certainty. It has been shown that, once such bids are introduced, the proposition that all successful bids must be made at or above the expected value of minority shares does not always hold. Bids below the expected value of minority shares may succeed, may be profitable, and may be used.

A crucial distinction that the analysis has highlighted is the difference in consequences between conditional bids and unconditional bids. While conditional bids below V_m cannot succeed, unconditional bids may. The power of unconditional bids results because they offer shareholders the prospect of having their shares acquired even if the bid fails. Because of shareholders' desire to have their shares acquired in such a case, a failure with certainty cannot be a rational expectations outcome of an unconditional bid. Thus, what might enable such a bid to succeed is the possibility that it will fail.

The model is consistent with two empirical observations—that bids sometimes fail (see Bradley, Desai, and Kim (1983)) and that the probability of a bid's success rises with the size of the bid's premium over the target's pre-bid price (see Walking (1985)). The first models of takeovers (e.g., Grossman and Hart (1980), Shleifer and Vishny (1986)) implied that all bids made will be successful. The inconsistency of this prediction with the evidence has called for constructing models in which some bids may fail, and such a model is offered in this

paper. (Other recent efforts to develop such a model include Bebchuk (1988), Hirshleifer and Titman (1988), and Jegadeesh and Chowdhry (1988)).

The most important testable implication of the model, however, is its prediction that bids below the expected value of minority shares may be made and may succeed. The model implies the presence of instances in which a bid succeeds even though the bid price is lower than the market price that unacquired shares have immediately following the successful bid's closing. If it is indeed found that there is a significant number of such instances, then such a finding will be inconsistent with the Grossman and Hart proposition and will provide evidence supporting the paper's model.⁴

Finally, although the Grossman and Hart conclusions concerning takeover bids must be refined in the way described in this paper, the main insights offered by their analysis do remain: the importance of expropriation in inducing bids and the importance of the expected value of minority shares in determining tender decisions and bid outcomes. In particular, although it was shown that bids may be made even in the absence of expropriation, the presence of expropriation may well be an important incentive to bidding.

Appendix

Proof of Proposition 4. Differentiating the right-hand side of (10) with respect to B gives

$$(A1) \quad \begin{aligned} W'(B) = & \sqrt{N/2\pi} I \sqrt{\tau(1-\tau)} e^{-1/2|q(\tau)|^2} [-q(\tau)] q'(\tau) \tau'(B) \\ & + \sqrt{N/2\pi} I \frac{1}{2\sqrt{\tau(1-\tau)}} (1-2\tau) e^{-1/2|q(\tau)|^2} \tau'(B). \end{aligned}$$

To get $q'(\tau)$, we differentiate both sides of (8) and get

$$(A2) \quad q'(\tau) = -\sqrt{N} \frac{\tau(1-k) + k(1-\tau)}{2[\tau(1-\tau)]^{3/2}}.$$

After substituting the right-hand side of (A2) for $q'(\tau)$ in (A1) and some algebraic manipulation, we get

$$(A3) \quad \begin{aligned} W'(B) = & \sqrt{N/2\pi} I (1/2) [\tau(1-\tau)]^{-3/2} \tau'(B) \\ & \{ (1/N)(1-2\tau)\tau(1-\tau) + (k-\tau)[\tau(1-k) + k(1-\tau)] \}. \end{aligned}$$

⁴ Note that the model should not be tested by whether, in successful bids, V_m exceeds B in general or even on average, but, rather, should be tested by whether there is a significant number of successful bids in which $V_m > B$. The Grossman and Hart proposition implies that B cannot be lower than V_m in any successful bid, and the proposition should thus be rejected if there is a significant set of successful bids in which $B < V_m$. That B is lower than V_m in a significant number of successful bids—rather than in all of them—is also what is implied by this paper's model. Recall that the model considers the case in which the raider can run the target more efficiently than current management (i.e., $V_t > V_0$). There are, however, cases in which the raider can run the target less efficiently than current management, but is nonetheless able to take over profitably due to the presence of expropriation (see Bagnoli and Lipman (1988) and Bebchuk (1988)). In this latter group of cases, which is not considered in this paper, V_m in successful bids is expected to be below B .

The sign of the right-hand side of (A3) depends on the sign of the sum in the braces (since all the terms outside the braces are positive). Since N is large, the value of the sum in braces (largely) depends on the second expression in the braces. And the second expression is positive for $\tau < k$ and negative for $\tau > k$. Thus, the optimal B is what (approximately) would produce a τ equal to k .

Finally, using the normal approximation, it follows that the probability of a takeover associated with $\tau = k$ is $1/2$. And, from (3), it follows that the B that will produce such τ and $Pr(\tau)$ is $(V_i + V_o)/2$.

Proof of Proposition 9. Consider the consequences of lowering a bid from V_m by an arbitrarily small ϵ . On the one hand, such lowering of the bid introduces a likelihood of $\epsilon/(V_m - V_o)$ that the bid will fail. Denoting by T_{nt} the expected number of tendered shares conditional on the bid's failure, the expected loss in the event that lowering the bid leads to the bid's failure is

$$(A4) \quad DN + T_{nt}(V_m - V_o - \epsilon) .$$

The first expression is the loss of the potential profit that a bid at V_m would produce, and the second expression is the expected loss on the shares that would have to be purchased through the failing bid.

On the other hand, the lower bid might also succeed with probability $(V_m - V_o - \epsilon)/(V_m - V_o)$, in which case lowering the bid would produce a saving of ϵ on each share purchased through the bid. Denoting by T_t the expected number of tendered shares in the event that the bid succeeds, the gain from lowering the bid in the event that the bid succeeds is

$$(A5) \quad T_t \epsilon .$$

Thus, a bid at $V_m - \epsilon$ will be superior to a bid at V_m if and only if

$$(A6) \quad \left(1 - \frac{\epsilon}{V_m - V_o}\right) T_t \epsilon > \left(\frac{\epsilon}{V_m - V_o}\right) [DN + (V_m - V_o - \epsilon) T_{nt}] ,$$

or, after rearranging terms, if and only if

$$(A7) \quad T_t - T_{nt} \frac{\epsilon}{V_m - V_o} > \frac{D}{V_m - V_o} N + \left(1 - \frac{\epsilon}{V_m - V_o}\right) T_{nt} .$$

Note that by setting ϵ sufficiently small, we can set the value of the right-hand side of (A7) as close as we wish to T_t and the value of the second expression on the right-hand side as close as we wish to T_{nt} . Note also that T_{nt} must, by definition, be lower than kN . And, finally, note that by setting ϵ sufficiently small, we can get the equilibrium probability of tendering τ to be as close to 1 as we wish, and thus also get T_t to be as close to N as we wish. Putting all of this together, it follows that a sufficient condition for (A7) to hold is that

$$(A8) \quad \frac{D}{V_m - V_o} < 1 - k ,$$

or, using the fact that $V_m - V_o = I - D$, that

$$(A9) \quad D < \frac{1-k}{2-k} I.$$

Proof of Proposition 10. If the bidder bids below $V_m - \alpha D$, its expected payoff will be as given by (18). Using the analysis of Proposition 4, the maximum value of the first expression on the right-hand side of (18) is $[Nk(1-k)/2\pi]^{1/2} I$ (a maximum that is obtained at $B = (V_m + V_o)/2$). And the second expression on the right-hand side of (18) must be smaller than $DN[\alpha D/(V_m - V_o)]$. Thus, the expected payoff to the bidder from any bid below $V_m - \alpha D$ cannot exceed

$$(A10) \quad \sqrt{Nk(1-k)/2\pi} (V_m - V_o) + DN \frac{V_m - \alpha D - V_o}{V_m - V_o}.$$

In contrast, if the bidder bids at V_m , its gain will be DN . Thus, a sufficient condition for the optimal bid to exceed $V_m - \alpha D$ is that DN exceed the right-hand side of (A10) and, thus, that

$$(A11) \quad DN \frac{\alpha D}{V_m - V_o} \geq \sqrt{Nk(1-k)/2\pi} (V_m - V_o).$$

And, after substituting $I - D$ for $V_m - V_o$ and rearranging terms, we get

$$(A12) \quad D \geq \frac{[k(1-k)/2\pi\alpha^2 N]^{1/4}}{1 + [k(1-k)/2\pi\alpha^2 N]^{1/4}} I.$$

References

- Bagnoli, M., and B. Lipman. "Successful Takeovers without Exclusion." *The Review of Financial Studies*, 1 (Spring 1988), 89-110.
- Bebchuk, L. "The Outcome of Takeover Bids under Alternative Private and Legal Arrangements." Working Paper, Harvard Law School (1988).
- Bradley, D.; A. Desai; and H. Kim. "The Rationale behind Interfirm Tender Offers: Information or Synergy?" *Journal of Financial Economics*, 11 (April 1983), 141-153.
- Feller, W. *An Introduction to Probability Theory and Its Applications*, 3rd ed. New York: John Wiley and Sons (1968).
- Fishman, M. "A Theory of Preemptive Takeover Bidding." *Rand Journal of Economics*, 19 (Spring 1988), 88-101.
- Grossman, S., and O. Hart. "Takeover Bids, the Free Rider Problem, and the Theory of the Corporation." *Bell Journal of Economics*, 11 (Spring 1980), 42-64.
- Harris, M., and A. Raviv. "Corporate Control Contests and Capital Structure." *Journal of Financial Economics*, 20 (Jan./March 1988), 55.
- Hirshleifer, D., and S. Titman. "Share Tendering Strategies and the Success of Hostile Takeover Bids." Working Paper, UCLA Graduate School of Management (1988).
- Jegadeesh, N., and B. Chowdhry. "Optimal Pre-Tender Offer Share Acquisition Strategy in Takeovers." Working Paper, UCLA Graduate School of Management (1988).
- Khanna, N. "Optimal Bidding for Tender Offers." Working Paper, The Univ. of Michigan Graduate School of Business (1988).
- Scharfstein, D. "The Disciplinary Role of Takeovers." *Review of Economic Studies*, 55 (April 1988), 185.
- Shleifer, A., and R. Vishny. "Large Shareholders and Corporate Control." *Journal of Political Economy*, 94 (April 1986), 461-488.
- Walking, R. "Predicting Tender Offer Success: A Logistic Analysis." *Journal of Financial and Quantitative Analysis*, 20 (Dec. 1985), 461-478.